WIKIPEDIA

Eastern Arabic numerals

The **Eastern Arabic numerals** (also called **Arabic–Hindu numerals**, **Arabic Eastern numerals** and **Indo-Persian numerals**) are the symbols used to represent the Hindu–Arabic numeral system, in conjunction with the Arabic alphabet in the countries of the Mashriq (the east of the Arab world), the Arabian Peninsula, and its variant in other countries that use the Perso-Arabic script in the Iranian plateau and Asia.

Eastern Arabic numerals on a clock in the Cairo Metro.

Contents

Origin
Other names
Numerals
Usage
Contemporary use
Notes
References

Clocks in the Ottoman Empire tended to use Eastern Arabic numerals.

Origin

The numeral system originates from an ancient Indian numeral system, which was re-introduced in the book *On the Calculation with Hindu Numerals* written by the medieval-era Iranian mathematician and engineer Khwarazmi,[1] whose name was Latinized as *Algoritmi*.[note 1]

Other names

These numbers are known as أرقام هندية ("Indian numbers") in Arabic. They are sometimes also called "Indic numerals" in English.[2] However, that is sometimes discouraged as it can lead to confusion with Indian numerals, used in Brahmic scripts of India.[3]

Numerals

Each numeral in the Persian variant has a different Unicode point even if it looks identical to the Eastern Arabic numeral counterpart. However the variants used with Urdu, Sindhi, and other South Asian languages are not encoded separately from the Persian variants. See U+0660 through U+0669 and U+06F0 through U+06F9.

Hindu-Arabic numerals	0	1	2	3	4	5	6	7	8	9
Eastern Arabic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Perso-Arabic variant	٠	١	٢	٣	۴	۵	۶	٧	٨	٩
Urdu variant	۰	۱	۲	۳	۴	۵	۶	۷	۸	۹

Usage

Written numerals are arranged with their lowest-value digit to the right, with higher value positions added to the left. That is identical to the arrangement used by Western texts using Hindu-Arabic numerals even though Arabic script is read from right to left. There is no conflict unless numerical layout is necessary, as is the case for arithmetic problems (as in simple addition or multiplication) and lists of numbers, which tend to be justified at the decimal point or comma.[4]

Contemporary use

Eastern Arabic numerals remain strongly predominant vis-à-vis Hindu-Arabic numerals in many countries to the East of the Arab world, particularly in Iran and Afghanistan.

In Arabic-speaking Asia as well as Egypt and Sudan both kinds of numerals are used alongside each other with Hindu-Arabic numerals numerals gaining more and more currency, now even in very traditional countries such as Saudi-Arabia.

In Pakistan, Hindu-Arabic Numerals are more extensively used as a considerable majority of the population is anglophone. Eastern numerals still continue to see use in Urdu publications and newspapers, as well as sign boards.

In North Africa (excluding Egypt and Sudan), only Hindu-Arabic numerals (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9) are now commonly used. In medieval times, these areas used a slightly different set (from which, via Italy, Western "Hindu-Arabic numerals" derive).

A bilingual Pakistani road sign showing the use of both Eastern Arabic and Hindu-Arabic numerals. The propensity towards Western Arabic numerals can be clearly seen.

Notes

1. Other Latin transliterations include *Algaurizin*.

References

1. https://en.wikipedia.org/wiki/Muhammad_ibn_Musa_al-Khwarizmi (https://en.wikipedia.org/wiki/Muhammad_ibn_Musa_al-Khwarizmi). Missing or empty `|title=` (help)
2. "Glossary of Unicode terms" (http://www.ibm.com/developerworks/library/glossaries

/unicode.html#indic_numerals). Retrieved 2 September 2015.
3. "Glossary" (https://www.unicode.org/glossary/). Retrieved 2 September 2015.
4. Menninger, Karl (1992). *Number words and number symbols: a cultural history of numbers* (https://books.google.com/books?id=YLJb6-OyUIQC&pg=PA415#v=onepage&q&f=false). Courier Dover Publications. p. 415. ISBN 0-486-27096-3.

Template:Sanskrit

WIKIPEDIA

Hindu–Arabic numeral system

The **Hindu–Arabic numeral system**[1] (also called the **Arabic numeral system** or **Hindu numeral system**)[2][note 1] is a positional decimal numeral system that is the most common system for the symbolic representation of numbers in the world. It was an ancient Indian numeral system which was re-introduced in the book *On the Calculation with Hindu Numerals* written by the medieval-era Iranian mathematician and engineer al-Khwarizmi, whose name was latinized as *Algoritmi*.[note 2][3] The system later spread to medieval Europe by the High Middle Ages.

The system is based upon ten (originally nine) different glyphs. The symbols (glyphs) used to represent the system are in principle independent of the system itself. The glyphs in actual use are descended from Brahmi numerals and have split into various typographical variants since the Middle Ages.

These symbol sets can be divided into three main families: Arabic numerals used in the Greater Maghreb and in Europe, Eastern Arabic numerals (also called "Indic numerals") used in the Middle East, and the Indian numerals used in the Indian subcontinent.

Eastern Arabic and Western Arabic numerals on a road sign in Abu Dhabi

Contents

Etymology

Positional notation

Symbols
- Glyph comparison

History
- Predecessors
- Development
- Adoption in Europe
- Adoption in East Asia
- Spread of the Western Arabic variant

See also

Notes

References

Bibliography

Etymology

The Hindu-Arabic or Indo-Persian numerals originated from India. Following its re-introduction in the book *On the Calculation with Hindu Numerals* written by the medieval-era Persian mathematician and engineer al-Khwarizmi, whose name was latinized

as *Algoritmi*,[3] it began to be referred to by the Persian and Arabic mathematicians as the "Hindu numerals" (where "Hindu" meant Indian). After its subsequent introduction to Europe, the Europeans referred to it as the "Arabic Numerals" based on a common misconception of associating the origin of oriental science with Arabic peoples.[4]

Positional notation

The Hindu–Arabic system is designed for positional notation in a decimal system. In a more developed form, positional notation also uses a decimal marker (at first a mark over the ones digit but now more usually a decimal point or a decimal comma which separates the ones place from the tenths place), and also a symbol for "these digits recur *ad infinitum*". In modern usage, this latter symbol is usually a vinculum (a horizontal line placed over the repeating digits). In this more developed form, the numeral system can symbolize any rational number using only 13 symbols (the ten digits, decimal marker, vinculum, and a prepended dash to indicate a negative number).

Although generally found in text written with the Arabic abjad ("alphabet"), numbers written with these numerals also place the most-significant digit to the left, so they read from left to right. The requisite changes in reading direction are found in text that mixes left-to-right writing systems with right-to-left systems.

Symbols

Various symbol sets are used to represent numbers in the Hindu–Arabic numeral system, most of which developed from the Brahmi numerals.

The symbols used to represent the system have split into various typographical variants since the Middle Ages, arranged in three main groups:

- The widespread Western "Arabic numerals" used with the Latin, Cyrillic, and Greek alphabets in the table, descended from the "West Arabic numerals" which were developed in al-Andalus and the Maghreb (there are two typographic styles for rendering western Arabic numerals, known as lining figures and text figures).
- The "Arabic–Indic" or "Eastern Arabic numerals" used with Arabic script, developed primarily in what is now Iraq. A variant of the Eastern Arabic numerals is used in Persian and Urdu.
- The Indian numerals in use with scripts of the Brahmic family in India and Southeast Asia. Each of the roughly dozen major scripts of India has its own numeral glyphs (as one will note when perusing Unicode character charts).

Glyph comparison

#	#	#	#	#	#	#	#	#	#	Script	See
0	1	2	3	4	5	6	7	8	9	Latin script	Arabic numerals
〇/零	一	二	三	四	五	六	七	八	九	East Asia	Chinese numerals, Japanese numerals, Korean numerals
ο/ō	Α'	Β'	Γ'	Δ'	Ε'	Ϛ'	Ζ'	Η'	Θ'	Modern Greek	Greek numerals
	א	ב	ג	ד	ה	ו	ז	ח	ט	Hebrew	Hebrew numerals
०	१	२	३	४	५	६	७	८	९	Devanagari	Indian numerals
૦	૧	૨	૩	૪	૫	૬	૭	૮	૯	Gujarati	
੦	੧	੨	੩	੪	੫	੬	੭	੮	੯	Gurmukhi	
๐	๑	๒	๓	๔	๕	๖	๗	๘	๙	Tibetan	
০	১	২	৩	৪	৫	৬	৭	৮	৯	Assamese / Bengali / Sylheti	Bengali-Assamese numerals
೦	೧	೨	೩	೪	೫	೬	೭	೮	೯	Kannada	
୦	୧	୨	୩	୪	୫	୬	୭	୮	୯	Odia	
൦	൧	൨	൩	൪	൫	൬	൭	൮	൯	Malayalam	
0	க	உ	ங	ச	ஞ	ண	எ	அ	ஆ	Tamil	Tamil numerals
0	౧	౨	౩	౪	౫	౬	౭	౮	౯	Telugu	
๐	๑	๒	๓	๔	๕	๖	๗	๘	๙	Khmer	Khmer numerals
๐	๑	๒	๓	๔	๕	๖	๗	๘	๙	Thai	Thai numerals
๐	๑	๒	๓	๔	๕	๖	๗	๘	๙	Lao	
ဝ	၁	၂	၃	၄	၅	၆	၇	၈	၉	Burmese	
٠	١	٢	٣	٤	٥	٦	٧	٨	٩	Arabic	Eastern Arabic numerals
۰	۱	۲	۳	۴	۵	۶	۷	۸	۹	Persian (Farsi) / Dari / Pashto	
۔	۱	۲	۳	۴	۵	۶	۷	۸	۹	Urdu / Shahmukhi	

As in many numbering systems, the numbers 1, 2, and 3 represent simple tally marks; 1 being a single line, 2 being two lines (now connected by a diagonal) and 3 being three lines (now connected by two vertical lines). After three, numbers tend to become more complex symbols (examples are the Chinese numerals and Roman numerals). Theorists believe that this is because it becomes difficult to instantaneously count objects past three.[5]

History

Predecessors

The Brahmi numerals at the basis of the system predate the Common Era. They replaced the earlier Kharosthi numerals used

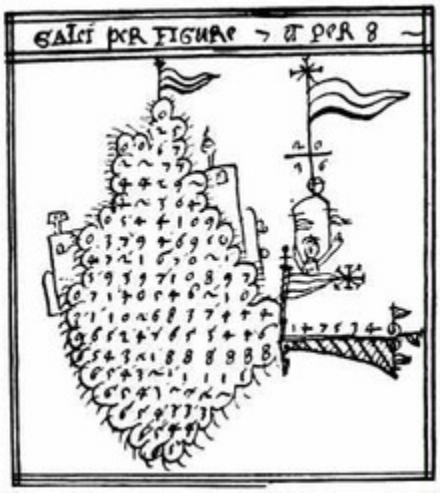
since the 4th century BC. Brahmi and Kharosthi numerals were used alongside one another in the Maurya Empire period, both appearing on the 3rd century BC edicts of Ashoka.[6]

Buddhist inscriptions from around 300 BC use the symbols that became 1, 4 and 6. One century later, their use of the symbols that became 2, 4, 6, 7 and 9 was recorded. These Brahmi numerals are the ancestors of the Hindu–Arabic glyphs 1 to 9, but they were not used as a positional system with a zero, and there were rather separate numerals for each of the tens (10, 20, 30, etc.).

The actual numeral system, including positional notation and use of zero, is in principle independent of the glyphs used, and significantly younger than the Brahmi numerals.

Development

The place-value system is used in the Bakhshali Manuscript. Although date of the composition of the manuscript is uncertain, the language used in the manuscript indicates that it could not have been composed any later than 400.[7] The development of the positional decimal system takes its origins in Hindu mathematics during the Gupta period. Around 500, the astronomer Aryabhata uses the word *kha* ("emptiness") to mark "zero" in tabular arrangements of digits. The 7th century *Brahmasphuta Siddhanta* contains a comparatively advanced understanding of the mathematical role of zero. The Sanskrit translation of the lost 5th century Prakrit Jaina cosmological text *Lokavibhaga* may preserve an early instance of positional use of zero.[8]


The "Galley" method of division.

These Indian developments were taken up in Islamic mathematics in the 8th century, as recorded in al-Qifti's *Chronology of the scholars* (early 13th century).[9]

The numeral system came to be known to both the Perso-Arabic mathematician Khwarizmi, who wrote a book, *On the Calculation with Hindu Numerals* in about 825, and the Arab mathematician Al-Kindi, who wrote four volumes, *On the Use of the Hindu Numerals* (كتاب في استعمال العداد الهندي [*kitāb fī isti'māl al-'adād al-hindī*]) around 830. These earlier texts did not use the Hindu numerals. Kushyar ibn Labban who wrote *Kitab fi usul hisab al-hind (Principles of Hindu Reckoning)* is one of the oldest surviving manuscripts using the Hindu numerals.[10] These books are principally responsible for the diffusion of the Hindu system of numeration throughout the Islamic world and ultimately also to Europe.

The first dated and undisputed inscription showing the use of a symbol for zero appears on a stone inscription found at the Chaturbhuja Temple at Gwalior in India, dated 876.[11]

In 10th century Islamic mathematics, the system was extended to include fractions, as recorded in a treatise by Syrian mathematician Abu'l-Hasan al-Uqlidisi in 952–953.[12]

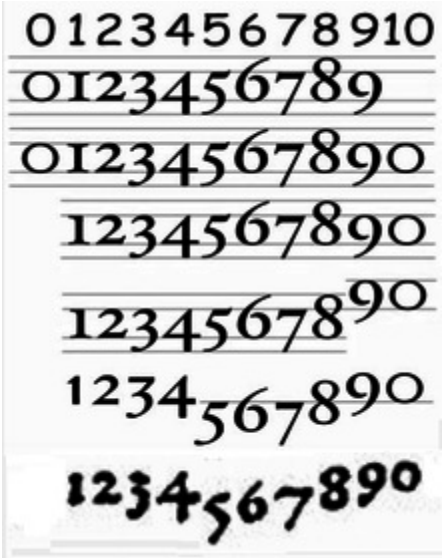
Adoption in Europe

In Christian Europe, the first mention and representation of Hindu-Arabic numerals (from one to nine, without zero), is in the *Codex Vigilanus*, an illuminated compilation of various historical documents from the Visigothic period in Spain, written in the year 976 by three monks of the Riojan monastery of San Martín de Albelda. Between 967 and 969, Gerbert of Aurillac discovered and studied Arab science in the Catalan abbeys. Later he obtained from these places the book *De multiplicatione et divisione* (*On multiplication and division*). After becoming Pope Sylvester II in the year 999, he introduced a new model of abacus, the so-called

Abacus of Gerbert, by adopting tokens representing Hindu-Arab numerals, from one to nine.

Leonardo Fibonacci brought this system to Europe. His book *Liber Abaci* introduced Arabic numerals, the use of zero, and the decimal place system to the Latin world. The numeral system came to be called "Arabic" by the Europeans. It was used in European mathematics from the 12th century, and entered common use from the 15th century to replace Roman numerals.[13][14]

The familiar shape of the Western Arabic glyphs as now used with the Latin alphabet (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) are the product of the late 15th to early 16th century, when they enter early typesetting. Muslim scientists used the Babylonian numeral system, and merchants used the Abjad numerals, a system similar to the Greek numeral system and the Hebrew numeral system. Similarly, *Fibonacci*'s introduction of the system to Europe was restricted to learned circles. The credit for first establishing widespread understanding and usage of the decimal positional notation among the general population goes to *Adam Ries*, an author of the German Renaissance, whose 1522 *Rechenung auff der linihen und federn* was targeted at the apprentices of businessmen and craftsmen.



The bottom row shows the numeral glyphs as they appear in type in German incunabula (Nicolaus Kesler, Basel, 1486)

Gregor Reisch, *Madame Arithmatica*, 1508

A calculation table, used for arithmetic using Roman numerals

Adam Ries, *Rechenung auff der linihen und federn*, 1522

Two arithmetic books published in 1514—*Köbel* (left) using a calculation table and *Böschenteyn* using numerals

Adam Ries, *Rechenung auff der linihen und federn* (2nd Ed.), 1525

Robert Recorde, *The ground of artes*, 1543

Peter Apian, *Kaufmanns Rechnung*, 1527

Adam Ries, *Rechenung auff der linihen und federn* (2nd Ed.), 1525

Adoption in East Asia

In AD 690, Empress Wu promulgated Zetian characters, one of which was "〇". The word is now used as a synonym for the number zero.

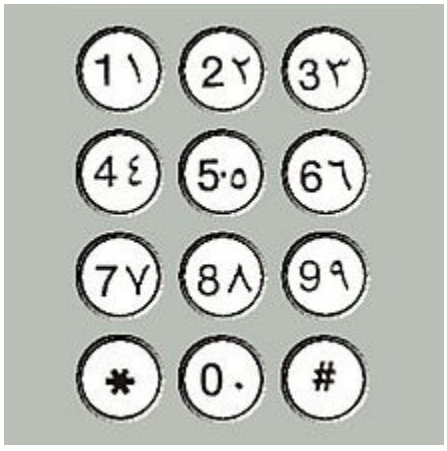
In China, Gautama Siddha introduced Hindu numerals with zero in 718, but Chinese mathematicians did not find them useful, as they had already had the decimal positional counting rods.[15][16]

In Chinese numerals, a circle (〇) is used to write zero in Suzhou numerals. Many historians think it was imported from Indian numerals by Gautama Siddha in 718, but some Chinese scholars think it was created from the Chinese text space filler "□".[15]

Chinese and Japanese finally adopted the Hindu–Arabic numerals in the 19th century, abandoning counting rods.

Spread of the Western Arabic variant

The "Western Arabic" numerals as they were in common use in Europe since the Baroque period have secondarily found worldwide use together with the Latin alphabet, and even significantly beyond the contemporary spread of the Latin alphabet, intruding into the writing systems in regions where other variants of the Hindu–Arabic numerals had been in use, but also in conjunction with Chinese and Japanese writing (see Chinese numerals, Japanese numerals).



An Arab telephone keypad with both the Western "Arabic numerals" and the Arabic "Arabic–Indic numerals" variants.

See also

- Arabic numerals
- Decimal
- Positional notation
- Numeral system
- History of mathematics
- 0 (number)

Notes

1. Hindu was the Persian name for "Indian" in the 10th century, when the Arabs adopted the number system. The connotation of "Hindu" as a religion was a later development.
2. Other Latin transliterations include *Algaurizin*.

References

- Flegg, Graham (2002). *Numbers: Their History and Meaning*. Courier Dover Publications. ISBN 0-486-42165-1.
- The Arabic numeral system – MacTutor History of Mathematics (http://www-history.mcs.st-and.ac.uk/HistTopics/Arabic_numerals.html)

1. David Eugene Smith and Louis Charles Karpinski, The Hindu–Arabic Numerals (http://www.gutenberg.org/etext/22599), 1911
2. William Darrach Halsey, Emanuel Friedman (1983). *Collier's Encyclopedia, with bibliography and index* (https://books.google.com/books?id=uIgxAQAAIAAJ&q=%22empire+was+expanding+and+contact+was+made+with+India%22&dq=%22empire+was+expanding+and+contact+was+made+with+India%22&hl=en&sa=X&ved=0ahUKEwi22ZabybvQAhWjLMAKHa9nAREQ6AEIGzAA).
3. Brezina, Corona (2006), *Al-Khwarizmi: The Inventor of Algebra* (https://books.google.com/books?id=3Sfrxde0CXIC&pg=PA39), The Rosen Publishing Group, pp. 39–40, ISBN 978-1-4042-0513-0, c.825)
4. Rowlett, Russ (2004-07-04), *Roman and "Arabic" Numerals* (http://www.unc.edu/~rowlett/units/roman.html), University of North Carolina at Chapel Hill, retrieved 2009-06-22
5. *Language may shape human thought* (https://www.newscientist.com/article.ns?id=dn6303), *New Scientist*, news service, Celeste Biever, 19:00 19 August 2004.
6. Flegg (2002), p. 6ff.
7. Pearce, Ian (May 2002). "The Bakhshali manuscript" (http://www-history.mcs.st-andrews.ac.uk/HistTopics/Bakhshali_manuscript.html). The MacTutor History of Mathematics archive. Retrieved 2007-07-24.
8. Ifrah, G. The Universal History of Numbers: From prehistory to the invention of the computer. John Wiley and Sons Inc., 2000. Translated from the French by David Bellos, E.F. Harding, Sophie Wood and Ian Monk
9. al-Qifti's *Chronology of the scholars* (early 13th century):
   > *... a person from India presented himself before the Caliph al-Mansur in the year 776 who was well versed in the siddhanta method of calculation related to the movement of the heavenly bodies, and having ways of calculating equations based on the half-chord [essentially the sine] calculated in half-degrees ... Al-Mansur ordered this book to be translated into Arabic, and a work to be written, based on the translation, to give the Arabs a solid base for calculating the movements of the planets ...*
10. Martin Levey and Marvin Petruck, Principles of Hindu Reckoning, translation of Kushyar ibn Labban Kitab fi usul hisab al-hind, p3, University of Wisconsin Press, 1965
11. Bill Casselman (February 2007). "All for Nought" (http://www.ams.org/featurecolumn/archive/india-zero.html). *Feature Column*. AMS.
12. Berggren, J. Lennart (2007). "Mathematics in Medieval Islam". *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook*. Princeton University Press. p. 518. ISBN 978-0-691-11485-9.
13. "Fibonacci Numbers" (http://www.halexandria.org/dward093.htm). *www.halexandria.org*.
14. Leonardo Pisano – page 3: "Contributions to number theory" (http://www.britannica.com/eb/article-4153/Leonardo-Pisano). Encyclopædia Britannica Online, 2006. Retrieved 18 September 2006.
15. Qian, Baocong (1964), *Zhongguo Shuxue Shi (The history of Chinese mathematics)*, Beijing: Kexue Chubanshe
16. Wáng, Qīngxiáng (1999), *Sangi o koeta otoko (The man who exceeded counting rods)*, Tokyo: Tōyō Shoten, ISBN 4-88595-226-3

Bibliography

- Menninger, Karl W. (1969). Number Words and Number Symbols: A Cultural History of Numbers. MIT Press. ISBN 0-262-13040-8.
- On the genealogy of modern numerals by Edward Clive Bayley (http://digital.nls.uk/early-gaelic-book-collections/pageturner.cfm?id=77845307)

WIKIPEDIA

Modern Arabic mathematical notation

Modern Arabic mathematical notation is a mathematical notation based on the Arabic script, used especially at pre-university levels of education. Its form is mostly derived from Western notation, but has some notable features that set it apart from its Western counterpart. The most remarkable of those features is the fact that it is written from right to left following the normal direction of the Arabic script. Other differences include the replacement of the Latin alphabet letters for symbols with Arabic letters and the use of Arabic names for functions and relations.

Contents

Features

Variations
- Numeral systems
- Mirrored Latin symbols

Examples
- Mathematical letters
- Mathematical constants and units
- Sets and number systems
- Arithmetic and algebra
- Trigonometric and hyperbolic functions
  - Trigonometric functions
  - Hyperbolic functions
  - Inverse trigonometric functions
  - Inverse hyperbolic functions
- Calculus
- Complex analysis

See also

References

External links

Features

- It is written from right to left following the normal direction of the Arabic script. Other differences include the replacement of the Latin alphabet letters for symbols with Arabic letters and the use of Arabic names for functions and relations.
- The notation exhibits one of the very few remaining vestiges of non-dotted Arabic scripts, as dots over and under letters (*i'jam*) are usually omitted.
- Letter cursivity (connectedness) of Arabic is also taken advantage of, in a few cases, to define variables using more than one letter. The most widespread example of this kind of usage is the canonical symbol for the radius of a circle **نق** (Arabic pronunciation: [nɑq]), which is written using the two letters nūn and qāf. When variable names are juxtaposed (as when expressing multiplication) they are written non-cursively.

Variations

Notation differs slightly from region to another. In tertiary education, most regions use the Western notation. The notation mainly differs in numeral system used, and in mathematical symbol used.

Numeral systems

There are three numeral systems used in right to left mathematical notation.

- "Western Arabic numerals" (sometimes called European) are used in western Arabic regions (e.g. Morocco)
- "Eastern Arabic numerals" are used in middle and eastern Arabic regions (e.g. Egypt and Syria)
- "Eastern Arabic-Indic numerals" are used in Persian and Urdu speaking regions (e.g. Iran, Pakistan, India)

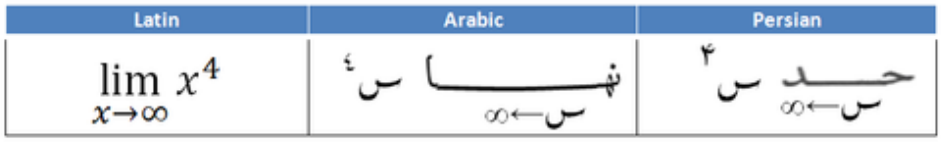
	0	1	2	3	4	5	6	7	8	9
European (descended from the West Arabic)	0	1	2	3	4	5	6	7	8	9
Arabic-Indic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Eastern Arabic-Indic (Persian and Urdu)	٠	١	٢	٣	۴	۵	۶	٧	٨	٩
Devanagari (Hindi)	०	१	२	३	४	५	६	७	८	९
Tamil		க	உ	ங	ச	ரு	சு	எ	அ	கூ

	0	1	2	3	4	5	6	7	8	9
European (descended from Western Arabic)	0	1	2	3	4	5	6	7	8	9
Arabic-Indic (Eastern Arabic)	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Perso-Arabic variant	٠	١	٢	٣	۴	۵	۶	٧	٨	٩
Urdu variant	٠	۱	۲	۳	۴	۵	۶	۷	۸	۹
Tamil variant		க	உ	ங	ச	ரு	சு	எ	அ	கூ

Written numerals are arranged with their lowest-value digit to the right, with higher value positions added to the left. That is identical to the arrangement used by Western texts using Hindu-Arabic numerals even though Arabic script is read from right to left. The symbols "٫" and "٬" may be used as the decimal mark and the thousands separator respectively when writing with Eastern Arabic numerals, e.g. ٣٫١٤١٥٩٢٦٥٣٥٨ *3.14159265358*, ١٬٠٠٠٬٠٠٠٬٠٠٠ *1,000,000,000*. Negative signs are written to the left of magnitudes, e.g. −٣ *−3*. In-line fractions are written with the numerator and denominator on the left and right of the fraction slash respectively, e.g. ٢/٧ *2/7*.

Mirrored Latin symbols

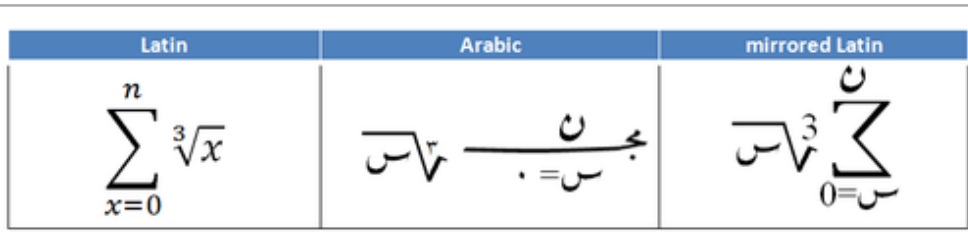
Sometimes, symbols used in Arabic mathematical notation differ according to the region:



Latin	Arabic	Persian
$\lim_{x\to\infty} x^4$	نهـــــا س٤ س←∞ [a]	حـــــد س٤ س←∞ [b]

- **^a** نهـا *nūn-hāʾ-ʾalif* is derived from the first three letters of Arabic نهاية *nihāya* "limit".
- **^b** حد *ḥadd* is Persian for "limit".

Sometimes, mirrored Latin symbols are used in Arabic mathematical notation (especially in western Arabic regions):



Latin	Arabic	Mirrored Latin
$\sum_{x=0}^{n} \sqrt[3]{x}$	مجـــــن س=٠ √٣س [c]	ن Σ س=0 √3س

- **^c** مجـ *mīm*-medial form of *ǧīm* is derived from the first two letters of Arabic مجموع *mağmūʿ* "sum".

However, in Iran, usually Latin symbols are used.

Examples

Mathematical letters

Latin	Arabic		Notes
a	ﺍ	ا	From the Arabic letter ا *ʾalif*; *a* and ا *ʾalif* are the first letters of the Latin alphabet and the Arabic alphabet's ʾabjadī sequence respectively
b	ﺏ	ب	A dotless ب *bāʾ*; *b* and ب *bāʾ* are the second letters of the Latin alphabet and the ʾabjadī sequence respectively
c	ﺟـ	ـــج	From the initial form of ج *ḥāʾ*, or that of a dotless ج *ǰīm*; *c* and ج *ǰīm* are the third letters of the Latin alphabet and the ʾabjadī sequence respectively
d	ﺩ	د	From the Arabic letter د *dāl*; *d* and د *dāl* are the fourth letters of the Latin alphabet and the ʾabjadī sequence respectively
x	ﺱ	س	From the Arabic letter س *sīn*. It is contested that the usage of Latin *x* in maths is derived from the first letter ش *šīn* (without its dots) of the Arabic word شيء *šayʾ (un)* [ʃajʔ(un)], meaning *thing*.[1] (*X* was used in old Spanish for the sound /ʃ/). However, according to others there is no historical evidence for this.[2][3]
y	ﺹ	ص	From the Arabic letter ص *ṣād*
z	ﻉ	ع	From the Arabic letter ع *ʿayn*

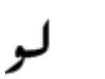
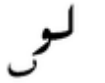
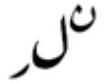
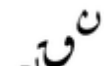
Mathematical constants and units

Description	Latin	Arabic		Notes
Euler's number	*e*	هـ	ه	Initial form of the Arabic letter ه *hāʾ*. Both Latin letter *e* and Arabic letter ه *hāʾ* are descendants of Phoenician letter 𐤄 *hē*.
imaginary unit	*i*	ت	ت	From ت *tāʾ*, which is in turn derived from the first letter of the second word of وحدة تخيلية *waḥdaẗun taḫīliyya* "imaginary unit"
pi	*π*	ط	ط	From ط *ṭāʾ*; also *π* in some regions
radius	*r*	نق	نق	From ن *nūn* followed by a dotless ق *qāf*, which is in turn derived from نصف القطر *nuṣfu l-quṭr* "radius"
kilogram	kg	كجم	كجم	From كجم *kāf-jīm-mīm*. In some regions alternative symbols like كغ (*kāf-ġayn*) or كغم (*kāf-ġayn-mīm*) are used. All three abbreviations are derived from كيلوغرام *kīlūġrām* "kilogram" and its variant spellings.
gram	g	جم	جم	From جم *jīm-mīm*, which is in turn derived from جرام *jrām*, a variant spelling of غرام *ġrām* "gram"
meter	m	م	م	From م *mīm*, which is in turn derived from متر *mitr* "meter"
centimeter	cm	سم	سم	From سم *sīn-mīm*, which is in turn derived from سنتيمتر "centimeter"
millimeter	mm	مم	مم	From مم *mīm-mīm*, which is in turn derived from مليمتر *millīmitr* "millimeter"
kilometer	km	كم	كم	From كم *kāf-mīm*; also كلم (*kāf-lām-mīm*) in some regions; both are derived from كيلومتر *kīlūmitr* "kilometer".
second	s	ث	ث	From ث *ṯāʾ*, which is in turn derived from ثانية *ṯāniya* "second"
minute	min	د	د	From د *dālʾ*, which is in turn derived from دقيقة *daqīqa* "minute"; also ق (ق , i.e. dotless ق *qāf*) in some regions
hour	h	س	س	From س *sīnʾ*, which is in turn derived from ساعة *sāʿa* "hour"
kilometer per hour	km/h	كم/س	كم/س	From the symbols for kilometer and hour
degree Celsius	°C	س°	س°	From س *sīn*, which is in turn derived from the second word of درجة سيلسيوس *darajat sīlsīūs* "degree Celsius"; also م° (م°) from م *mīmʾ*, which is in turn derived from the first letter of the third word of درجة حرارة مئوية "degree centigrade"
degree Fahrenheit	°F	ف°	ف°	From ف *fāʾ*, which is in turn derived from the second word of درجة فهرنهايت *darajat fahranhāyt* "degree Fahrenheit"
millimeters of mercury	mmHg	ممز	ممز	From ممز *mīm-mīm zayn*, which is in turn derived from the initial letters of the words مليمتر زئبق "millimeters of mercury"
Ångström	Å	أ	أ	From أ *ʾalif* with *hamzah* and ring above, which is in turn derived from the first letter of "Ångström", variously spelled أنغستروم or أنجستروم

Sets and number systems

Description	Latin	Arabic		Notes
Natural numbers	$\mathbb{N}$		ط	From ط *ṭāʾ*, which is in turn derived from the first letter of the second word of عدد طبيعي *ʿadadun ṭabīʿiyyun* "natural number"
Integers	$\mathbb{Z}$		ص	From ص *ṣād*, which is in turn derived from the first letter of the second word of عدد صحيح *ʿadadun ṣaḥīḥun* "integer"
Rational numbers	$\mathbb{Q}$		ن	From ن *nūn*, which is in turn derived from the first letter of نسبة *nisba* "ratio"
Real numbers	$\mathbb{R}$		ح	From ح *ḥāʾ*, which is in turn derived from the first letter of the second word of عدد حقيقي *ʿadadun ḥaqīqiyyun* "real number"
Imaginary numbers	$\mathbb{I}$		ت	From ت *tāʾ*, which is in turn derived from the first letter of the second word of عدد تخيلي *ʿadadun taẖīliyyun* "imaginary number"
Complex numbers	$\mathbb{C}$		م	From م *mīm*, which is in turn derived from the first letter of the second word of عدد مركب *ʿadadun markabun* "complex number"
Empty set	$\varnothing$	$\varnothing$	∅	
Is an element of	$\in$	$\ni$	∋	A mirrored ∈
Subset	$\subset$	$\supset$	⊃	A mirrored ⊂
Superset	$\supset$	$\subset$	⊂	A mirrored ⊃
Universal set	$\mathbf{S}$		ش	From ش *šīn*, which is in turn derived from the first letter of the second word of مجموعة شاملة *majmūʿatun šāmila* "universal set"

Arithmetic and algebra

Description	Latin	Arabic		Notes
Percent	%	٪	٪	e.g. *100%* "١٠٠٪"
Permille	‰	٪٪	٪٪	٪٪ is an Arabic equivalent of the per ten thousand sign ‱.
Is proportional to	$\propto$	∝	∝	A mirrored ∝
n th root	$\sqrt[n]{}$		ⁿ√	ں is a dotless ن *nūn* while ⁿ√ is a mirrored radical sign √
Logarithm	$\log$		لو	From لو *lām-wāw*, which is in turn derived from لوغاريتم *lūġārītm* "logarithm"
Logarithm to base b	$\log_b$		لوب	
Natural logarithm	$\ln$		لوه	From the symbols of logarithm and Euler's number
Summation	$\sum$		مجـ	مجـ *mīm*-medial form of *jīm* is derived from the first two letters of مجموع *majmūʿ* "sum"; also ⅀ (⅀, a mirrored summation sign ∑) in some regions
Product	$\prod$		جـذ	From جذ *jīm-ḏāl*. The Arabic word for "product" is جداء *jadāʾun*. Also ∏ in some regions.
Factorial	$n!$		ں!	Also ں! (!ں) in some regions
Permutations	${}^nP_r$		ر(ن ، ر)ل	Also ل(ن ، ر) ((ر ،ن)ل) is used in some regions as $\mathbf{P}(n,r)$
Combinations	${}^nC_k$		ك(ن ، ك)ق	Also ق(ن ، ك) ((ك ،ن)ق) is used in some regions as $\mathbf{C}(n,k)$ and $\binom{n}{k}$ ($\binom{ن}{ك}$) as the binomial coefficient

Trigonometric and hyperbolic functions

Trigonometric functions

Description	Latin	Arabic		Notes
Sine	**sin**	**حا**	حا	from حاء *ḥāʾ* (i.e. dotless ج *jīm*)-ـٰـا *ʾalif*; also **جب** (جب *jīm-bāʾ*) is used in some regions (e.g. Syria); Arabic for "sine" is جيب *jayb*
Cosine	**cos**	**حتا**	حتا	from حتا *ḥāʾ* (i.e. dotless ج *jīm*)-*tāʾ*-ـٰـا *ʾalif*; also **تجب** (تجب *tāʾ-jīm-bāʾ*) is used in some regions (e.g. Syria); Arabic for "cosine" is جيب تمام
Tangent	**tan**	**طا**	طا	from طا *ṭāʾ* (i.e. dotless ظ *ẓāʾ*)-ـٰـا *ʾalif*; also **ظل** (ظل *ẓāʾ-lām*) is used in some regions (e.g. Syria); Arabic for "tangent" is ظل *ẓill*
Cotangent	**cot**	**طتا**	طتا	from طتا *ṭāʾ* (i.e. dotless ظ *ẓāʾ*)-*tāʾ*-ـٰـا *ʾalif*; also **تظل** (تظل *tāʾ-ẓāʾ-lām*) is used in some regions (e.g. Syria); Arabic for "cotangent" is ظل تمام
Secant	**sec**	**فا**	فا	from ما dotless ق *qāf*-ـٰـا *ʾalif*; Arabic for "secant" is أو قاطع
Cosecant	**csc**	**فتا**	فتا	from متا dotless ق *qāf-tāʾ*-ـٰـا *ʾalif*; Arabic for "cosecant" is أو قاطع تمام

Hyperbolic functions

The letter **ز** (ز *zayn*, from the first letter of the second word of دالة زائدية "hyperbolic function") is added to the end of trigonometric functions to express hyperbolic functions. This is similar to the way **h** is added to the end of trigonometric functions in Latin-based notation.

Latin	sinh	cosh	tanh	coth	sech	csch
Arabic	حاز	حتاز	طاز	طتاز	فاز	فتاز

Description	Hyperbolic sine	Hyperbolic cosine	Hyperbolic tangent	Hyperbolic cotangent	Hyperbolic secant	Hyperbolic cosecant
Latin	**sinh**	**cosh**	**tanh**	**coth**	**sech**	**csch**
Arabic	حاز	حتاز	طاز	طتاز	فاز	فتاز

Inverse trigonometric functions

For inverse trigonometric functions, the superscript ١− in Arabic notation is similar in usage to the superscript **−1** in Latin-based notation.

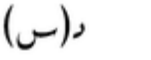
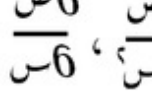
Latin	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$	$\cot^{-1}$	$\sec^{-1}$	$\csc^{-1}$
Arabic	حا-١	حتا-١	طا-١	طتا-١	فا-١	فتا-١

Description	Inverse sine	Inverse cosine	Inverse tangent	Inverse cotangent	Inverse secant	Inverse cosecant
Latin	$\mathbf{sin^{-1}}$	$\mathbf{cos^{-1}}$	$\mathbf{tan^{-1}}$	$\mathbf{cot^{-1}}$	$\mathbf{sec^{-1}}$	$\mathbf{csc^{-1}}$
Arabic	حا-١	حتا-١	طا-١	طتا-١	فا-١	فتا-١

Inverse hyperbolic functions

Description	Inverse hyperbolic sine	Inverse hyperbolic cosine	Inverse hyperbolic tangent	Inverse hyperbolic cotangent	Inverse hyperbolic secant	Inverse hyperbolic cosecant
Latin	$\sinh^{-1}$	$\cosh^{-1}$	$\tanh^{-1}$	$\coth^{-1}$	sech^{-1}	csch^{-1}
Arabic	حاز$^{-١}$	حتاز$^{-١}$	طاز$^{-١}$	طتاز$^{-١}$	فاز$^{-١}$	فتاز$^{-١}$

Calculus

Description	Latin	Arabic		Notes
Limit	$\lim$		نهـــا	نهـــا *nūn-hāʾ-ʾalif* is derived from the first three letters of Arabic نهاية *nihāya* "limit"
function	$\mathbf{f}(x)$		د(س)	د *dāl* is derived from the first letter of دالة "function". Also called تابع, تا for short, in some regions.
derivatives	$\mathbf{f}'(x), \frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{\partial y}{\partial x}$		د'(س)، $\frac{دص}{دس}$ ، $\frac{د^٢ص}{دس^٢}$ ، $\frac{∂ص}{∂س}$	' is a mirrored prime ′ while ، is an Arabic comma. The ∂ signs should be mirrored: ∂.
Integrals	$\int, \iint, \iiint, \oint$		∫، ∬، ∭، ∮	Mirrored ∫, ∬, ∭ and ∮

Complex analysis

Latin	Arabic
$z = x + iy = r(\cos\varphi + i\sin\varphi) = re^{i\varphi} = r\angle\varphi$	
	ع = س + ت ص = ل(حتا ى + ت حا ى) = ل هـتـى = ل ∠ ى

See also

- Mathematical notation
- Arabic Mathematical Alphabetic Symbols

References

1. Moore, Terry. "Why is X the Unknown" (http://www.ted.com/talks/terry_moore_why_is_x_the_unknown.html). Ted Talk.
2. Cajori, Florian. *A History of Mathematical Notation* (https://books.google.com/books?id=7juWmvQSTvwC&pg=PA382&redir_esc=y#v=onepage&q&f=false). Courier Dover Publications. pp. 382–383. Retrieved 11 October 2012. "Nor is there historical evidence to support the statement found in Noah Webster's Dictionary, under the letter x, to the effect that 'x was used as an abbreviation of Ar. shei (a thing), something, which, in the Middle Ages, was used to designate the unknown, and was then prevailingly transcribed as xei.'"
3. *Oxford Dictionary, 2nd Edition*. "There is no evidence in support of the hypothesis that x is derived ultimately from the mediaeval transliteration xei of shei "thing", used by the Arabs to denote the unknown quantity, or from the compendium for L. res "thing" or radix "root" (resembling a loosely-written x), used by mediaeval mathematicians."

External links

- Multilingual mathematical e-document processing (https://www.ima.umn.edu/materials/2006-2007/SW12.8-9.06/2289/MathArabIMAe.pdf)
- Arabic mathematical notation (http://www.w3.org/TR/arabic-math/) - W3C Interest Group Note.
- Arabic math editor (http://www.wiris.com/ar_sa/editor/demo/) - by WIRIS.
